Algebraic Functions in Geometry

1. Consider an equilateral triangle.

   a) Express the area A of an equilateral triangle as a function of the length of its side, s. (Students should memorize this formula)

   b) Find the area when the side is 3 centimeters.

   c) If the equilateral triangle is rotated about the altitude, find the volume of the resulting solid in terms of s, the length of a side.

   d) Find the volume when the side is 3 cm.

   e) If the equilateral triangle is rotated about a side, find the volume of the resulting solid in terms of s.

   f) Find the volume when the side is 3 cm.

2. In a right triangle, the length of the hypotenuse is 5 units, x and y represent the lengths of the legs.

   a) Sketch a picture of the problem situation.

   b) Write an equation for the problem situation using the Pythagorean theorem.

   c) The graph of the equation above represents a _____. (The answer is NOT a triangle.)

   d) What is the domain for x and the range for y in this problem situation?

   e) Solve for y for this problem situation only. What do the coordinate pairs on this function represent?

   f) Give the coordinates of the points on the graph in terms of x. Sketch.

   g) In which quadrant are the answers for this problem situation? ___

   h) If $x = 3$, find y.

   i) If $x = 4.8$, find y.

   j) If one leg of the right triangle is $y = 3.758$, find the length of the other leg, x.

3. The portion of the vertical line through the point $(x, 0)$ that lies between the x-axis and the graph of $y = \sqrt{x}$ is revolved about the x-axis.

   a) Sketch the problem situation. Give the coordinates on the graph in terms of x.

   b) Express the area A of the resulting disk (i.e. circle) as a function of x.

   c) Find the area of the disk if $x = 9$.

4. Rotate the region bounded by the x-axis, the graph of $y = 3x$, and the vertical line that passes through the point $(0, x)$ around the x-axis.

   a) Sketch the problem situation. Give the coordinates of a point on the line in terms of x.

   b) Let C be the cone formed by rotating the area of the triangle under the line $y = 3x$ from 0 to x about the x-axis. Express the volume of C in terms of y.

   c) Find the volume when $x = 4$

   d) The same enclosed area is revolved around the y-axis, determine the volume of the resulting solid in terms of x.

   e) Find the volume when $x = 4$.

5. Triangle OAB is an isosceles triangle with vertex 0 at the origin and vertices A in quadrant I and B in quadrant II on the parabola $y = 9 - x^2$.

   a) Sketch the problem situation. Give the coordinates of points on the graph in terms of x.

   b) Express the area of the triangle as a function of x.

   c) Give the domain for x in this problem situation.

   d) Find the area when $x = 2$.